Stats 2 - January 2013

(1) a) From calc:

$\sum x = 318.36$ $\qquad \bar{x} = 53.06$

$\sum x^2 = 16898.6816$ $\qquad s = 1.140175...$

$n = 6$, $v = 5$ $\qquad$ need t as don't know σ

2 tailed, 95% for t: ± 2.571

$\therefore$ 95% CI: $53.06 \pm 2.571 \times \dfrac{1.140175}{\sqrt{6}}$

$= 53.06 \pm 1.1965...$

$= (51.86, 54.26)$

b) 1) Sample mean (53.06) is lower than last years (53.41), so claim may be true

2) BUT, 53.41 lies with confidence interval, so no strong evidence that the mean is different.

(2) a) H_0: no association between property type and time to sell

H_1: there is an association between type and time

Expected

	Flat	T	S-D	D
< 3	8.736	34.944	24.192	16.128
> 3	4.264	17.056	11.808	7.872

Flat < 5, so combine Flat + Terrace

New Expected

	F+T	S-D	D
< 3	43.68	24.192	16.128
> 3	21.32	11.808	7.872

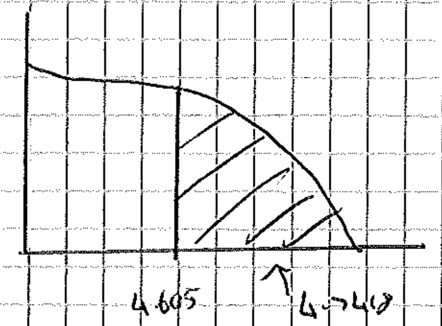
$\chi^2 \leftarrow \dfrac{(O-E)^2}{E}$ $\qquad \sum \chi^2 = 4.7418$

	F+T	S-D	D
< 3	0.7386	0.5946	0.2173
> 3	1.5132	1.2281	0.4452

Test Statistic: $\sum X^2 = 4.7418$

$\nu = (3-2) \times (2-1) = 2$

Critical Value: 10% $X^2(2) = 4.605$



$4.7418 > 4.605$

$\therefore$ Reject H_0

Evidence at 10% level suggest association between property type & time to sell

b) i) More terraces than any other so may have big effect

ii) Very far away from their expected values for selling time.

③ a) $WD \sim Po(1.5)$

i) $P(WD = 3) = \frac{e^{-1.5} \times 1.5^3}{3!} = 0.126$

ii) $WE \sim Po(0.5)$

Over both days, $Both \sim Po(1)$

$P(Both > 1) = 1 - P(Both \leq 1)$

$= 1 - 0.7358 = 0.264$

iii) $Week \sim Po(8.5)$

$P(Week < 10) = P(Week \leq 9)$

$= 0.653$ (from tables)

b) $Week \sim Po(8.5)$

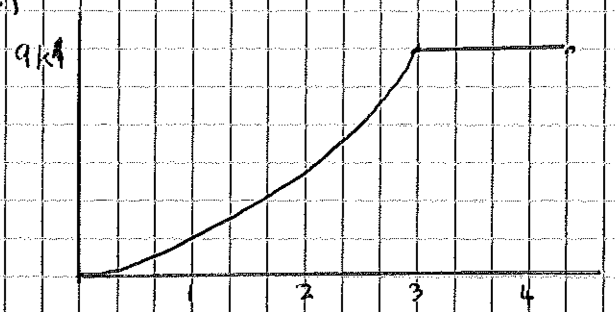
From tables, $P(Week \leq 15) = 0.9862$

$P(Week \leq 16) = 0.9934$

$\therefore$ He should keep 16 tubes

c) Not consistent use of light (and $\therefore$ failure) over the course of a day

④ a)



b) $\int_0^3 kx^2\,dx = \Big[$

$= 27k/3 - 0 = 9k$

Area between 3 & 4 $= 1 \times 9k$
$= 9k$

$\therefore$ Total Area $= 9k + 9k = 18k$

Area must $= 1 \rightarrow k = 1/18$

c) i) $1/2$ area is to left of 3 $\rightarrow$ median $= 3$

ii) $\int_0^Q kx^2\,dx = \left[kx^3/3\right]_0^Q = \left[x^3/54\right]_0^Q = 0.25$

$Q^3/54 - 0 = 0.25$

$\rightarrow Q^3 = 13.5 \rightarrow Q = \sqrt[3]{13.5}$

$= 2.3811..$

⑤ a) $E(X) = 0 \times 0.1 + 1 \times 0.35 + 2 \times 0.25 + 3 \times 0.2 + 4 \times 0.1$

$= 1.85 =$ mean

$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.35 + 2^2 \times 0.25 + 3^2 \times 0.2 + 4^2 \times 0.1$

$= 4.75$

$\therefore Var(X) = E(X^2) - [E(X)]^2$

$= 4.75 - 1.85^2 = 1.3275$

b) i) $T = c + nX$

ii) $E(T) = c + n\,E(X)$

$= c + 1.85n$

$Var(T) = n^2\,Var(X)$

$= 1.3275\,n^2$

⑥ a) $P(t \leq x) = 0.9 \Rightarrow F(x) = 0.9$

$\rightarrow \frac{t^3}{216} = 0.9$

$\rightarrow t^3 = 194.4$

$\rightarrow t = 5.7929..$ weeks

$5.7929 \times 7 = 40.55 \rightarrow 41$ days

b) Must differentiate $F(t)$

$$\frac{t^3}{216} \rightarrow \frac{3t^2}{216} = \frac{t^2}{72}$$

$$\therefore \quad f(t) = \begin{cases} t^2/72 & 0 \le t \le 6 \\ 0 & \text{otherwise} \end{cases}$$

c) $$E(T) = \int_0^6 t\left(\frac{t^2}{72}\right) dx = \int_0^t \frac{t^3}{72} dx$$

$$= \left[\frac{t^4}{288}\right]_0^6 = \frac{6^4}{288} - 0 = 4.5$$

$$E(T^2) = \int_0^6 t^2\left(\frac{t^2}{72}\right) dx = \int \frac{t^4}{72} dx$$

$$= \left[\frac{t^5}{360}\right]_0^6 = \frac{6^5}{360} - 0 = 21.6$$

$$\therefore \ Var(T) = E(T^2) - [E(T)]^2$$

$$= 21.6 - 4.5^2 = 1.35$$

d) $sd(T) = \sqrt{Var(T)} = \sqrt{1.35} = 1.16189...$

$$P(T > (4.5 + 1.16189)) = P(T > 5.662)$$

$$= 1 - P(T < 5.662)$$

$$= 1 - F(5.662)$$

$$= 1 - \frac{5.662^3}{216} = 0.160 \ (3sf)$$

7) a) $H_0: \mu = 20 \leftarrow (3020 - 20)$

$H_1: \mu \neq 20$ (2 tailed test)

$$\bar{y} = \frac{1847}{100} = 18.47$$

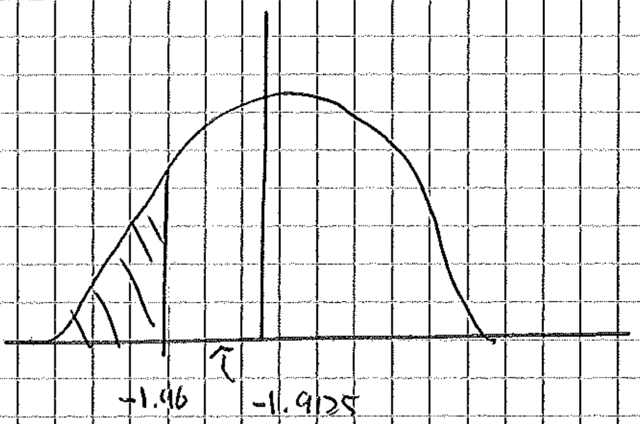
$$s^2 = \frac{6336}{99} = 64$$

$$n = 100$$

As $n > 30$, we can use Z because of CLT.

Test Statistic: $z = \dfrac{18.47 - 20}{\sqrt{64/100}} = -1.9125$

Critical Value: 5%, 2 tailed $\rightarrow$ $z = \pm 1.96$



$-1.9125 > -1.96$

$\therefore$ Accept H_0

No significant evidence at 5% level that the mean value has changed

b) No error as we accepted H_0!